

TOLERANCE MODELLING OF THERMOMECHANICS IN MICROSTRUCTURED MEDIA

Piotr Ostrowski

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Contents

Preface	5
Nomenclature	9
1 Preliminaries	11
1.1 Normed and Banach spaces.....	11
1.2 Linear operators and functional variations.....	13
1.3 Inner product and Hilbert spaces.....	16
2 Microstructured solid	18
2.1 Overview.....	20
2.2 Periodic solids.....	21
2.3 Heteroperiodic solids.....	22
3 Problem formulation	24
3.1 Linearised heat conduction in microstructured solids.....	24
3.1.1 Direct description.....	25

3.1.2	Variational approaches.....	26
3.2	Linearised thermoelasticity in microstructured solids	30
3.2.1	Direct description	32
3.2.2	Variational approach.....	34
3.3	Nonlinear heat conduction in microstructured solids.....	37
3.3.1	Direct description	37
4	Tolerance averaging technique	39
4.1	Basic concepts and definitions	40
4.2	Averaging of tolerance periodic functions.....	46
5	Selected problems: tolerance modelling	56
5.1	Linearised heat conduction in microstructured solids.....	56
5.1.1	Extended stationary action principle.....	58
5.1.2	Time-localized stationary action principle	62
5.1.3	Orthogonalisation method.....	65
5.2	Linearised thermoelasticity in microstructured solids	68
5.2.1	Extended stationary action principle.....	70
5.2.2	Orthogonalisation method.....	75
5.3	Nonlinear heat conduction in microstructured solids.....	80
5.4	Summary.....	83
5.4.1	Initial-boundary conditions.....	83
5.4.2	Summary and error estimation.....	86
6	Selected problems: asymptotic modelling	88
6.1	Linearised heat conduction in microstructured solids.....	88
6.1.1	Consistent asymptotic modelling	89
6.1.2	Tolerance-asymptotic modelling.....	98
6.2	Linearised thermoelasticity in microstructured solids	100
6.2.1	Consistent asymptotic modelling	100
6.2.2	Tolerance-asymptotic modelling.....	108
6.3	Nonlinear heat conduction in microstructured solids.....	109
6.3.1	Tolerance-asymptotic modelling.....	110
6.4	Summary.....	111
7	Examples of application	113
7.1	Linearised heat conduction in microstructured solids	113
7.2	Linearised thermoelasticity in microstructured media.....	123
	Bibliography	133
	Streszczenie	140

Preface

This book is a complex study of tolerance averaging technique, cf. Woźniak and Wierzbicki (2000), as a mathematical tool for the description of physical processes in microstructured solids. Based on the definitions of tolerance periodic, slowly varying and highly oscillating functions, adopted from Woźniak et al. (2010), fundamental averaging properties and assertions are proved by unified notations. Some of them are similar to those from Woźniak and Wierzbicki (2000) and Woźniak et al. (2008) but this time they are expressed in the language of Hilbert spaces, and by different averaging operators, what makes this result novelty. These properties are given together with estimated tolerance parameters, neglected in the process of modelling, so the truncation error could be eventually estimated.

Another novelty contained in this book is a step-by-step derivation of averaged model equations, oriented towards linearised thermoelasticity and linear and nonlinear heat conduction in microstructured media. As well as variational as direct approach is here presented together with its assumptions and function spaces. An estimation of tolerance parameter in all methods is explicitly exposed, what is the necessity in quantity evaluation of obtained results.

The last chapter is filled with numerous examples illustrating averaged equations applied to the specific micro-structure. Solutions for few initial-boundary value problems compared with other known models and solutions.

This monograph would not be finished or even exist if not an inspiration and motivation from the people of my surround. I would like to thank Bohdan Michalak, who introduced me first to the scientific world and tolerance modelling, and who led me patiently through the years as my truly mentor. Also many thanks I send to Jarosław Jędrysiak, who very often threw me challenges by giving me fine example and deadlines. I have to mention Izabela Kazubska, who never gave up in motivating me and supporting my person on my way to science duty fulfilment, for which I am grateful. I address my gratitude to Ewelina Pazera. Thanks to her I was

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